PHYSICAL REVIEW D **67**, 047301 (2003)

Viscous cosmological models and accelerated universes

G. M. Kremer* and F. P. Devecchi†

Departamento de Física, Universidade Federal do Paraná, Caixa Postal 19044, 81531-990, Curitiba, Brazil

(Received 6 November 2002; revised manuscript received 19 November 2002; published 28 February 2003)

It is shown that a present acceleration with a past deceleration is a possible solution to the Friedmann equation by considering the Universe as a mixture of a scalar with a matter field and by including a nonequilibrium pressure term in the energy-momentum tensor. The dark energy density decays more slowly with respect to the time than the matter energy density does. The inclusion of the nonequilibrium pressure leads to a less pronounced decay of the matter field with a shorter period of past deceleration.

DOI: 10.1103/PhysRevD.67.047301 PACS number(s): 98.80.Cq

According to recent cosmological observations there exists evidence that the Universe is flat (see Sievers *et al.* [1]) and expanding with a positive acceleration (see Perlmutter *et al.* [2] and Riess *et al.* [3]). The flatness of the Universe is connected with the total density parameter $\Omega_{\rm tot}$ which is the sum of the density parameters related to the vacuum energy Ω_Λ, cold dark matter $\Omega_{\rm CDM}$, and baryons Ω_b, i.e., $\Omega_{\rm tot}=\Omega_\Lambda+\Omega_{\rm CDM}+\Omega_b$. The sum of the density parameters of cold dark matter and baryons refers to the matter density parameter $\Omega_m=\Omega_{\rm CDM}+\Omega_b$. The most recent values for the density parameters from measurements of the anisotropy of the cosmic microwave background (CMB) are [1]

$$\Omega_{\rm tot}=1.01^{+0.09}_{-0.06}, \quad \Omega_m=0.37\pm 0.11,$$

and

$$\Omega_b=0.060\pm 0.020.$$

Measurements of the redshift of the type Ia supernova SN 1997ff indicate a present acceleration of the Universe and, according to Turner and Riess [4], provide a past decelerating phase of the Universe.

Phenomenological cosmologies play an important role in the understanding of the evolution of the Universe. A remarkable combination of general relativity and thermodynamics allows the description of different regimes in cosmological theories. Models that include the recent experimental evidence of a present accelerated Universe therefore deserve increasing attention.

On the other hand, there exist other regimes where a positive acceleration is present. Indeed, in inflationary models an exponential expansion with positive acceleration for the early Universe is proposed where a hypothetical particle, the inflation, is at the core of that mechanism. These ideas play the role of a precursor for the relation between vacuum (or dark) energy and a cosmological constant that depends on time. For recent reviews on dark energy one is referred to the works by Peebles and Ratra [5] and Turner [6].

What we know today is that dark energy dominates the composition of our Universe and is responsible for the positive accelerated expansion. It interacts weakly with ordinary matter (baryonic matter represents at maximum 5% of the whole mass/energy composition of our Universe).

The point we would like to discuss in this work is that a past decelerated with a present accelerated expansion of the Universe can be found as a solution of the Friedmann equation. The Universe is modeled as a mixture of two constituents, namely, a scalar field that represents the dark energy and a matter field that describes the baryonic matter and the cold dark matter; in our approach the interaction between the different constituents refers only to the one between the dark energy and the matter via the gravitational field which is represented by the cosmic scale factor $a(t)$. We model this interaction by using the ideas of a thermodynamic theory, by means of the inclusion of a nonequilibrium pressure term in the energy-momentum tensor which represents an irreversible process of energy transfer between the matter and the gravitational fields. Within the framework of first order Eckart thermodynamic theory the nonequilibrium pressure is proportional to the Hubble parameter and its proportionality factor is identified with the bulk viscosity, so that this model is referred to in the literature as a viscous cosmological model (see, for example, Ref. [7]).

Although the transition from a past deceleration to a present acceleration of the Universe is found as a possible solution of the Friedmann equation independently of the presence of the nonequilibrium pressure term, it is very questionable to model the Universe as a perfect gas mixture of scalar and matter fields evolving without dissipative effects (for further discussions on this subject one is referred to Ref. [8]). In this work, it is shown that the dark energy density decays more slowly with respect to the time than the matter energy density does. These conclusions are valid for both cases where the nonequilibrium pressure is present or absent. However, the inclusion of a nonequilibrium pressure leads to a less pronounced decay of the matter field with a shorter period of past deceleration, since the nonequilibrium pressure is the responsible for the energy transfer between the matter and the gravitational fields.

We shall consider a spatially flat, homogeneous and isotropic Universe described by the Robertson-Walker metric. With respect to the four-velocity U^μ the energy-momentum tensor $T^{\mu\nu}$ is decomposed as[1]

*Email address:kremer@fisica.ufpr.br

†Email address: devecchi@fisica.ufpr.br

[1]Units have been chosen so that $c=1$.

$$T^{\mu\nu}=(\rho_X+\rho_m+p_X+p_m+\varpi)U^\mu U^\nu-(p_X+p_m+\varpi)g^{\mu\nu}, \tag{1}$$

where p_X is the pressure of the dark energy, p_m is the pressure of the matter field, and ϖ denotes the nonequilibrium pressure which is coupled to the irreversible process of energy transfer between matter and gravitational fields.

In a comoving frame the conservation law of the energy-momentum tensor $T^{\mu\nu}{}_{;\nu}=0$ leads to the balance equation for the energy density

$$\dot{\rho}_X+\dot{\rho}_m+3H(\rho_X+\rho_m+p_X+p_m+\varpi)=0, \tag{2}$$

where $H=\dot{a}/a$ is the Hubble parameter and the dot refers to a differentiation with respect to time.

The identification of the dark energy with a scalar field allow us to assume that the balance equation for the dark energy density is given by

$$\dot{\rho}_X+3H(\rho_X+p_X)=0. \tag{3}$$

Hence, the balance equation for the dark energy density decouples from that for the energy density of the matter and we have from Eqs. (2) and (3)

$$\dot{\rho}_m+3H(\rho_m+p_m)=-3H\varpi. \tag{4}$$

The equation that connects the evolution of the cosmic scale factor $a(t)$ with the energy densities of the scalar and matter fields is given by the Friedmann equation

$$H^2=\frac{8\pi G}{3}(\rho_X+\rho_m), \tag{5}$$

where G is the gravitational constant.

In a previous work [9] we have calculated the energy-momentum pseudotensor of the gravitational field in a flat Robertson-Walker metric and found

$$T_G^{00}=-\frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2. \tag{6}$$

If we identify the T_G^{00} with the energy density ρ_G of the gravitational field we can obtain the relationship $\rho_G=-(\rho_X+\rho_m)$ thanks to the Friedmann equation (5).

Now we differentiate $\rho_G=-(\rho_X+\rho_m)$ with respect to the time and by using the balance equation for the energy density (4) we get

$$\dot{\rho}_G+3H(\rho_G-p_m-p_X)=3H\varpi, \tag{7}$$

which can be interpreted as a balance equation for the energy density of the gravitational field. Moreover, if we compare Eqs. (4) and (7) we infer that the right-hand side of both equations has a term (with opposite sign) proportional to the nonequilibrium pressure which is responsible to the transfer of energy between the gravitational and matter fields.

It is usual to assume for the pressure of the dark energy the equation of state $p_X=w_X\rho_X$, with $w_X<-1/3$ (see Peebles and Ratra [5]). Hence, one can obtain by integration of Eq. (3)

$$\frac{\rho_X}{\rho_X^0}=\left(\frac{a_0}{a}\right)^{3(w_X+1)}, \tag{8}$$

where ρ_X^0 and a_0 are the values of the dark energy density and of the cosmic scale factor at $t=0$ (by adjusting clocks), respectively.

In order to determine the time evolution of the dark energy density one has to know the evolution of the cosmic scale factor which is determined from the system of Eqs. (4), (5), and (8). This system of equations is closed by assuming a relationship between the pressure and the energy density of the matter and a constitutive equation for the nonequilibrium pressure ϖ. We assume a barotropic equation of state for the pressure $p_m=w_m\rho_m$, with $0\leqslant w_m\leqslant 1$ and a linear relationship between the nonequilibrium pressure ϖ and the Hubble parameter H within the framework of Eckart first order thermodynamic theory

$$\varpi=-3\eta H, \quad \eta=\alpha(\rho_X+\rho_m). \tag{9}$$

In the above equation η is the coefficient of bulk viscosity which is consider to be proportional to the energy density of the mixture and α is a constant.

We differentiate the Friedmann equation (5) with respect to the time and get by the use of Eqs. (8) and (9)

$$\dot{H}=\frac{3}{2}\left[\frac{(w_m-w_X)}{1+\rho_m^0/\rho_X^0}\left(\frac{1}{a}\right)^{3(w_X+1)}+(3\alpha H-w_m-1)H^2\right]. \tag{10}$$

Above ρ_m^0 is the energy density of the matter field at $t=0$ (by adjusting clocks). Moreover, all terms in Eq. (10) are dimensionless quantities defined by

$$H\equiv\frac{H}{H_0}, \quad t\equiv tH_0, \quad a\equiv\frac{a}{a_0}, \quad \alpha\equiv\alpha H_0,$$

with

$$H_0=\sqrt{\frac{8\pi G}{3}(\rho_X^0+\rho_m^0)}. \tag{11}$$

Equation (10) is a second-order differential equation for the cosmic scale factor $a(t)$ which is a function of four parameters, namely, w_m, w_X, α, and ρ_m^0/ρ_X^0. The solution of the differential equation (10) is found by specifying values for two initial conditions and for the four parameters.

Instead of using the constitutive equation (9) for the nonequilibrium pressure one may consider it as a variable within the framework of extended (causal or second-order) thermodynamic theory. In this case the evolution equation for the nonequilibrium pressure—in a linearized theory—reads (see, for example, Ref. [10])

$$\varpi+\tau\dot{\varpi}=-3\eta H, \tag{12}$$

where τ is a characteristic time. Here we follow the works [11] and assume that the characteristic time is given by $\tau = \eta/\rho$.

Hence, we obtain from the Friedmann equation (5) and from the evolution equation for the nonequilibrium pressure (12) the following system of differential equations:

$$\dot{H} = \frac{3}{2}\left[\frac{(w_m - w_X)}{1+\rho_m^0/\rho_X^0}\left(\frac{1}{a}\right)^{3(w_X+1)} - (w_m+1)H^2 + \varpi\right], \tag{13}$$

$$\varpi + \alpha\dot{\varpi} = -3\alpha H^3, \tag{14}$$

thanks to the second term in Eq. (9). Apart from the dimensionless quantities introduced above, $\varpi \equiv 8\pi G\varpi/(3H_0^2)$ is a dimensionless nonequilibrium pressure.

If we specify values for three initial conditions and for the four parameters w_m, w_X, α, and ρ_m^0/ρ_X^0 we can obtain from the system of Eqs. (13) and (14) the time evolution of the cosmic scale factor $a(t)$ and of the nonequilibrium pressure $\varpi(t)$.

Once $a(t)$ is determined from Eq. (10) or from the system (13) and (14) one can find the energy densities by using the expressions

$$\rho_X = \left(\frac{1}{a}\right)^{3(w_X+1)}, \quad \rho_m = \left(1 + \frac{\rho_X^0}{\rho_m^0}\right)H^2 - \frac{\rho_X^0}{\rho_m^0}\left(\frac{1}{a}\right)^{3(w_X+1)}, \tag{15}$$

where $\rho_X \equiv \rho_X/\rho_X^0$ and $\rho_m \equiv \rho_m/\rho_m^0$ are dimensionless quantities.

The initial conditions we choose at the instant of time $t = 0$ (by adjusting clocks) are $a(0) = 1$ for the cosmic scale factor and $H(0) = 1$ for the Hubble parameter and $\dot{\varpi}(0) = 0$ for the nonequilibrium pressure. Since the differential equations (10), (13), and (14) do depend on the four parameters w_m, w_X, α, and ρ_m^0/ρ_X^0 there exists much freedom to find their solutions. In order to obtain the graphics in Fig. 1 we have chosen (a) $w_X = -0.7$ so that the condition $w_X < -1/3$ holds, (b) $w_m = 0.2$, so that $0 < w_m < 1/3$ where the value $w_m = 0$ refers to dust and $w_m = 1/3$ to radiation, (c) $\rho_m^0/\rho_X^0 = 2$, i.e., the amount of the matter energy density is twice that of the dark energy density, (d) $\alpha = 0$ when the nonequilibrium pressure is absent and $\alpha = 0.05$ (say) when it is present.

We have plotted in Fig. 1 the dark energy density ρ_X, the matter energy density ρ_m and the acceleration $\ddot{a}$ as functions of the time t for the three cases: (a) by using a second-order thermodynamic theory (straight lines), (b) by using a first-order thermodynamic theory (dashed lines), and (c) by neglecting the nonequilibrium pressure (perfect fluid solution). We infer from this figure that for the parameters chosen above the behavior of the solutions of the first-order and second-order thermodynamic theories is practically the same. In the three cases there exist periods where the Universe is decelerating and accelerating. Moreover, the dark energy density decays more slowly than the matter energy density does. Even when a small amount of dark energy density—with respect to the matter energy density—is taken into account, these fields evolve in such a manner that for large times the amount of dark energy density is very large with respect to the energy density of the matter field. The inclusion of the nonequilibrium pressure leads to (i) a slower decay of the matter field with respect to the time, which is a consequence of the irreversible process of energy transfer between the matter and gravitational fields, (ii) a smaller interval of past deceleration, and (iii) a more rapid decay of the dark energy density.

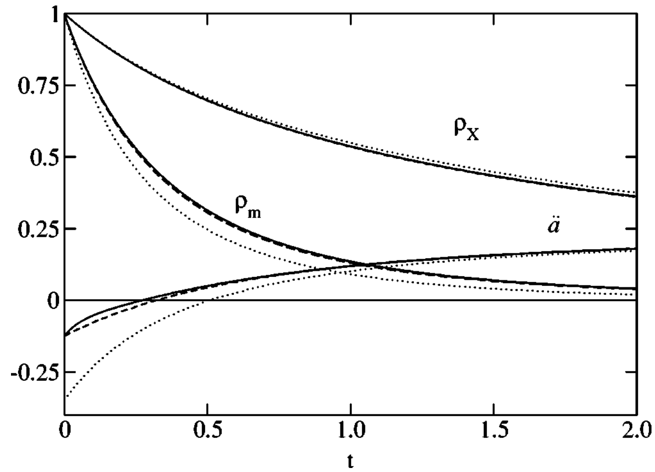

FIG. 1. Dark energy density ρ_X, matter energy density ρ_m, and acceleration $\ddot{a}$ vs time t for perfect fluid (dotted line) and first-order (dashed line) and second-order (straight line) thermodynamic theories.

Other conclusions can be obtained by changing the values of the parameters. In the case where there is no dark energy density $\rho_X = 0$ there is no period of acceleration, i.e., only a period of deceleration is possible. By changing the interval w_m to $1/3 < w_m < 2/3$—which corresponds to the interval between radiation $w_m = 1/3$ and nonrelativistic matter $w_m = 2/3$—the period of past deceleration increases. This last result is also found when w_X increases, i.e., for $w_X > -0.7$. By increasing the value of the dimensionless constant α the effect of the nonequilibrium pressure is more pronounced and the period of past deceleration decreases.

[1] J.L. Sievers *et al.*, astro-ph/0205387.
[2] S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999); S. Perlmutter, M.S. Turner, and M. White, Phys. Rev. Lett. **83**, 670 (1999).
[3] A.G. Riess *et al.*, Astrophys. J. **560**, 49 (2001).
[4] M.S. Turner and A.G. Riess, Astrophys. J. **569**, 18 (2002).
[5] P.J.E. Peebles and B. Ratra, astro-ph/0207347.
[6] M.S. Turner, astro-ph/0207297.
[7] G.L. Murphy, Phys. Rev. D **8**, 4231 (1973); Ø. Grøn, Astrophys. Space Sci. **173**, 191 (1990); V. Romano and D. Pavón,

Phys. Rev. D **47**, 1396 (1993); A.A. Coley and R.J. van den Hoogen, Class. Quantum Grav. **12**, 1977 (1995); W. Zimdahl, Phys. Rev. D **61**, 083511 (2000); G.M. Kremer and F.P. Devecchi, *ibid.* **65**, 083515 (2002).

[8] E. Calzetta and M. Thibeault, Phys. Rev. D **63**, 103507 (2001); D. Pavón, L.P. Chimento, and A.S. Jakubi, astro-ph/0210038; W. Zimdahl, A.B. Balakin, D.J. Schwarz, and D. Pavón, astro-ph/0210204.

[9] G.M. Kremer and F.P. Devecchi, Phys. Rev. D **66**, 063503 (2002).

[10] I. Müller and T. Ruggeri, *Rational Extended Thermodynamics* (Springer, New York, 1998); C. Cercignani and G.M. Kremer, *The Relativistic Boltzmann Equation: Theory and Applications* (Birkhäuser, Basel, 2002).

[11] V.A. Belinskii, E.S. Nikomarov, and I.M. Khalatnikov, Sov. Phys. JETP **50**, 213 (1979); A. Di Prisco, L. Herrera, and J. Ibáñez, Phys. Rev. D **63**, 023501 (2000).